CURVATURE DUCTILITY OF REINFORCED CONCRETE BEAMS

Shaker A. El-Behairy
Professor
Ain Shams University
Cairo, Egypt

Hamdy H. Shaheen
Professor
Building Research Center
Cairo, Egypt

Gamal I. Khaleel
Lecturer
Benha higher institute of Tech.
Benha, Egypt

Ahmed I. Abdel Mouti
Lecturer Assistant
Benha higher institute of Tech.
Benha, Egypt

ABSTRACT

Ductility may be defined as the ability to under-go deformation without a substantial reduction in the capacity of the member[4]. The curvature ductility factor is a fare more meaningful index for ductility demand that is because once plastic hinge has commerced in a structure, the deformations concentrated at the plastic hinge positions by rotation of the plastic hinge. The fundamental information needed by the designer concerns the required member section behavior at the plastic hinge expressed by the curvature ductility factor and its variation with the type of steel used and the ratio of the compression reinforcement.

The study of the flexural ductility was carried out mainly through a series of tests on simply supported beams under pure bending. The principal test variables were the compression reinforcement ratio and the type of steel reinforcement [1].

INTRODUCTION.

there are many measures to assess ductility[2,3]. One of the most common measures is the ductility factor which expressed in terms of deflection [5] or curvature. Curvature ductility factor is a far more meaningful index for ductility demand compared with the deflection ductility factor.

It needs to be recognized that there can be a significant difference between the magnitude of the deflection and the curvature ductility factor. This is because once plastic hinging has commenced in a structure, the deformations concentrated at the plastic hinge positions and further displacement occurs mainly by rotation of the plastic hinges. Thus, the required curvature ductility factor will be greater than the deflection ductility factor. So, our interest here will be on the curvature ductility.

TEST PROGRAM AND EXPERIMENTAL INVESTIGATION

a- Experimental Test Program:

The test program consists of 3 groups of reinforced concrete beams, as shown in table (1). These groups contain 12 reinforced concrete beams with rectangular cross section (15x25 cm), the clear span is 260 cm. The tested beams cover the studied parameters that affect the curvature ductility of reinforced concrete beams which are:, the type of the tension reinforcement (normal mild steel, high strength steel and cold worked steel), the ratio of the compression reinforcement and the types of the compression reinforcement.

b. Test Apparatus and Instrumentation.

Loading and measuring devices:

Vertical steel frame stiffened by two link members with a mechanical screw jack were used to apply the required loads on the beams as shown in fig.(1), (2).

The applied load from the (16 ton) mechanical screw jack was measured using two load cells each of maximum capacity (5 ton). Rotations were measured over the supports of the beams using Klinometer , which is supported on the beam using a special device to ensure its fixity with the beam. On the other hand, the deflection and steel strain were also measured.

c. Moment - Curvature Diagrams.

Ductility may be defined according to the curvature of the section. The fundamental information needed by the designer concerns the required member section behaviour at the plastic hinge expressed by the maximum curvature ductility factor ($\phi u/\phi y$), or by the final curvature ductility factor ($\phi f/\phi y$) where :

ϕu = The maximum curvature of the section.

ϕy = The curvature of the section at the point of first yield of steel.

ϕ_f = The curvature of the section when the maximum moment decreases to 80% of its value along the descending branch of the Moment - Curvature diagram.

It is found to be very difficult to measure the curvature directly by measuring the elongation and the contraction of the concrete due to the spalling of the compressive block after a certain value of strain. So the rotation of the beam over the support is measured using the Klinometer device which measures the angle of rotation of the beam.

d. Calculation of Curvature From The Rotation Measurements.

The maximum curvature of the section can be calculated using the rotation measurement as follows:

It is known that

$$\theta = \int \phi \, dx \qquad \ldots\ldots (1)$$

From the above equation, it is understandable that by knowing the rotation, it becomes possible to determine the curvature of the section.

Figure (3) shows the curvature diagrams for a beam with two concentrated loads. From the figure the following can be derived:

For $M \le My$

$$\phi = 1.29 * \theta a \qquad \ldots\ldots (2)$$

For $My < M \le Mu$

$$\theta a = \theta ae + \theta ap \qquad \ldots\ldots (3)$$

where θae and θap are the values of the elastic and plastic rotation respectively.

$$\phi = \phi y * (M/My) + [(2\,\theta a - 1.55*\phi y*(M/My)) / (1.55 - 1.05*(My/M))] \qquad \ldots\ldots (4)$$

where $\phi y = 1.29\,\theta ay$

For $Mu < M \le Mp$

Where Mp = The final bending moment.

$$\phi = 4*\theta a - 3.1*\phi y*(Mu/My) - 2*(\phi u - \phi y*(Mu/My))*(1.55-1.05*(My/Mu)) + \phi u \qquad \ldots\ldots (5)$$

where $\phi u = \phi y*(Mu/My) + [(2*\theta a - 1.55*\phi y*(Mu/My)) / (1.55-1.05*(My/Mu))]$

From the above equations, the curvature can be calculated at each increment of moment and then the moment curvature diagram for every group of beams can be investigated.

ANALYSIS OF THE EXPERIMENTAL RESULTS

Based on the experimental results, the behaviour of the test specimens is discussed in terms of the moment - curvature relationships.

a. Moment - Curvature Diagrams.

Figure (4) shows Moment - Curvature diagram for beams of group (A). The figure indicates that the three beams have the same elastic behaviour up to the point of first yield of steel. It is also indicated that by increasing the compression reinforcement ratio, the maximum curvature increases up to (As'/As=0.75). The Curvature decreased again, because by increasing the compression reinforcement ratio, the bond stress between the concrete and the steel increases, up to a point where the bond stress becomes more than the bond strength between the concrete and the steel then, the bond failure occurs. It shows also that at high values of compression reinforcement ratio, the slope of the descending portion of the moment - curvature diagrams descends steeper than that of the low values of compression reinforcement ratio. This result is understandable as, at low values of compression reinforcement ratio and after the first yield of tension steel, the compression reinforcement begins to yield and buckle causing a decrease in the beam carrying capacity, where the concrete compressive strain remains below the maximum value. So, with the increase of curvature, the stress of the concrete compressive block decreases gradually causing a gradual decrease in the load carrying capacity of the beam. On the other hand with the high values of compression reinforcement ratio, the yielding and buckling of the compression reinforcement occurs with a high value of concrete compressive strain. Therefore, with the increase of curvature the carrying load decreases significantly.

Figure (5) shows Moment - Curvature diagrams for beams of group (B). From the figure, it is indicated that the behaviour of the four beams is similar up to the point of first yield of steel. After that, the ultimate moment increases with the increase of the compression reinforcement ratio. Such increase can be explained as, after reaching the point of first yield of steel, the steel strength begins to increase again with the increase of strain due to the strain hardening of this type of steel as mentioned before. Accordingly the ultimate moment increases. It is also indicated that by increasing the compression reinforcement ratio, the slop of the Moment - Curvature diagram along the descending branch increases (the reason for this increase was discussed before in group A).

Figure (6) shows a comparison of Moment - Curvature diagrams for beams of group (C). It indicates that with the increase of the compression reinforcement ratio, the maximum curvature increases up to a point where a sudden drop off occurs in the load due to a sudden cut off in the tension reinforcement. This cut off in the tension reinforcement is understandable as the steel strain increases with the increase of deflection. As the cold worked steel possesses a little elongation (about 11 %), so a sudden cut off is occurred at a certain value of deflection.

b. <u>The Curvature Ductility Factor.</u>

Ductility factor may be defined as: the maximum ductility factor which is the ratio between the curvature at the point of maximum moment and the curvature at the point of first yield of steel; or, as the final ductility factor, which is the ratio between the final curvature (the curvature when the moment decreases to 80% of its ultimate value along the descending branch of the moment - curvature diagram) and the curvature at the point of first yield of steel. Referring to table (2) and with the help of the previous figures, the following results can be derived:

Group (A).

For group (A), it is indicated that by increasing the compression reinforcement ratio (As/As'), from 0.25 to 0.5 the maximum ductility factor increases 17 time, where as, for (AS'/As) more than .75 the maximum ductility factor decreases again due to the bond failure, as this plain bars are relatively week against the bond failure; or special precautions should be made to prevent this type of failure.

In case of the final ductility factor, it is indicated from table (2) that with low values of the compression reinforcement ratio, a significant increase is maintained in the ratio between the final ductility factor and the maximum ductility factor. In case of (As'/As =.25) the final ductility factor is about 7.7 times the maximum ductility factor, where as in case of (Á =.5), the final ductility factor is found to be 1.04 times the maximum ductility factor respectively. From the above two values, it is obvious that in case of low values of (As'/As), the gain in the ductility factor, by decreasing the load carrying capacity to 80% of the ultimate moment along the descending branch of the moment - curvature diagram, is significantly high. However in using high values of the compression reinforcement ratio, no significant increase in the ductility factor is gained due to a decrease of moment carrying capacity to 80 % of the ultimate moment.

Group (B).

In group (B), where the beams are reinforced using high tensile steel, it is indicated that by increasing the compression reinforcement ratio up to (As'/As =.42), the maximum ductility factor increases to about 3.2 times that of (As'/As =0.25). Where as by increasing the compression reinforcement ratio to (As'/As =.67), the maximum ductility factor increases by about 8.33 times that of (As'/As=.25). On the other hand, by increasing the compression reinforcement ratio up to (As'/As= 1.0), the maximum ductility factor increases to 12 times that of (As'/As =.25). From the above it can be noticed that, for this type of steel, by increasing the compression reinforcement ratio,the maximum ductility factor increases proportionally.

In case of the final ductility factor, it is obvious, from table (2), that the ratios of the final ductility factors to the maximum ductility factors are about (3.43 , 1.71 , 1.8 , 1.56) for (As'/As =0.25, 0.42, 0.67, 1.0)respectively . These values indicate that by increasing the compression reinforcement ratio, the gain in the ductility due to the decrease of the ultimate moment by about 80% along the descending branch of the moment - curvature diagram decreases for this type of steel.

Group (C).

In case of group (C), where the beams are reinforced using the cold worked steel, it is indicated that by increasing the compression reinforcement ratio from (As'/As =.25) to (As'/As =.42) the maximum ductility increases to about 2.2 times. While the ratio for (As'/As =.67) is found to be 3.8 times that of (As'/As =.25). On the other hand, by increasing the compression reinforcement ratio up to (As'/As=1.0), the maximum ductility is found to be 3.6 times that of (As'/As=.25). From the above results it can be noticed that by increasing the compression reinforcement, ratio a relatively small increase in the maximum ductility factor is maintained. This result is understandable as this type of steel possesses little ductility. So, with an increase of strain more than a certain value, the tension reinforcement is to be cut off and no more curvature is maintained.

In case of the final ductility factor, it is found that the final ductility factors for beams with (As'/As =.25 & .42) are approximately the same. On the other hand, the ratio of the final ductility factors to the maximum ductility factors are (2.43 & 1.28) for beams with (As'/As =.25 & .42) respectively. For beams with (As'/As =.67 & 1.0), after reaching the ultimate moment, a sudden cut off occurs, resulting in a dramatical decrease of moment. So no value for the final ductility factor is maintained. From the above results, it is not practically useful to increase the compression reinforcement ratio to high values.

c. **Comparison of Curvature Among The Beams of Groups (A),(B)&(C).**

Figure (7) shows the relationship between the deflection at the point of first yield of steel and the compression reinforcement ratio (As'/As) for groups (A) , (B) & (C). It indicates that the deflection at the first yield of steel decreases with the increase of the compression reinforcement ratio. Besides, the beams of group (A), which are reinforced using normal mild steel have the least curvature at the point of first yield of steel relative to these of the other two groups, where no significant deference is maintained between these of the groups B & C.

d. **Comparison of Moment - Curvature Diagrams For Beams of Groups (A),(B)&(C).**

Figure (8), (9), (10) & (11) show a comparison of moment - curvature diagrams for the beams of the three groups. Figure (8) indicate that:
The slope of the moment - curvature diagram along the descending branch for beam BA-1 is found to be the flattest among the three beams, which results in an increase in the final ductility factor for this beam relative to the other two beams (the ratio of the final ductility factor for beam BA-1 to that of beams BB-1 & BC-1 is about 2.0 & 2.70 respectively). Where as, figure (9) indicates that the maximum curvature for beam BB-2 is found to be about 1.25 that of beam BC-2 on the other hand along the descending branch of the moment - curvature diagrams, it is found that the slope of the curve

is flatter for beam BB-2 than that for beam BC-2. Thus, results in an increase in the final ductility factor for beam BB-2 relative to that of beam BC-2, which can be refereed to the little elongation of the cold-worked steel. Moreover, figure (10) shows that the value of the maximum moment for beam BB-3 is found to be higher than that of beam BC-3 due to the strain hardening of the high tensile steel as explained before. It is also found that the maximum curvature for beam BB-3 is about 3 times that of beam BC-3. Where as along the descending branch of the moment - curvature diagram, it is found that the moment decreases suddenly for beam BC-3 due to a sudden cut off in the tension reinforcement, that leads to no value for the final ductility factor for this beam. Finally, figure (11) shows the maximum moment and the maximum curvature increases for beam BB-4 relative to the other two beams due to the strain hardening of this type of steel. On the other hand, along the descending branch of the moment - curvature diagram, a sudden drop off in the maximum moment occurs for beams BA-4 & BC-4 as bond failure occurs for beam BA-4. Then a sudden cut off occurs in the tension reinforcement for beam BC-4 as previously discussed.

e. Comparison of Ductility Factors For Groups (A),(B)& (C).

Figure (12) shows the relationship between the maximum ductility factors and the compression reinforcement ratio. It indicates that at low ratios of the compression reinforcement, there is no significant difference in the maximum ductility factors among the three beams up to a point where (As'/As =.3). By increasing the compression reinforcement ratio, the ductility factors for the beams of group (A) increase dramatically and more than these of the other two groups . Yet at high values of the compression reinforcement ratio, a special precautions should be made to prevent the bond failure that may occure when using plain bars (normal mild steel). On the other hand, for the beams of group (C), it is indicated that by increasing the compression reinforcement ratio, the maximum ductility factor increases very slightly due to the low elongation value for the cold worked steel as previously discussed. The increase of the maximum ductility factor for beams of group (A) relative to that of group (B) can be discussed as, for the same ratio of the tension reinforcement, with the increase of curvature the steel stress remains equal to (fy) for group (A) while, in beams of group (B), the steel stress increases with the increases of the deflection which leads to an increase in the neutral axis depth and hence a decrease in the ductility factor relative to that in group (A).

CONCLUSION

From the experimental results the following conclusion can be achieved.

- for beams of group (A) :

by increasing the compression reinforcement ratio (As'/As), from (0.25) up to (0.5 & 1.00), the maximum curvature ductility factor increases to (2.62 & 7.7) times.

-For beams of group (B)
by increasing the compression reinforcement ratio (As'/As), from (0.25) up to (0.42, .64 & 1.00), the maximum curvature ductility factor increases to (3.2, 8.3 & 12.1). It is also found that the ratios between the final curvature ductility factor and the maximum curvature ductility factor are (2.7, 1.55, 1.22 &1.08) for (As'/As) equal to (0.25, .42, .64 & 1.00).

-For beams of group (C)
by increasing the compression reinforcement ratio (As'/As), from (0.25) up to (0.42, .64 & 1.00), the maximum curvature ductility factor increases to (2.1, 3.8 & 3.70). It is also found that the ratios between the final curvature ductility factor and the maximum curvature ductility factor are (2.9, 1.60) for (As'/As) equal to (0.25, .42).

-It is found also that there is no significant effect for the compression reinforcement on the behavior of the beams up to the point of first yield of steel.

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Table (1) Reinforced Concrete Beams Description

Group No.	Beam No.	Ccu kg/cm²	Tension R.F.T		Compression R.F.T		P'/P	Stirrups /m'
			As	Fy	As'	Fy		
A	BA-1	385	3 ø 16	3.2 t/cm²	2 ø 10	3.2 t/cm²	0.26	10 ø 6
	BA-2		3 ø 16		2ø12+1ø10		0.5	
	BA-3		3 ø 16		3ø12+1ø10		0.753	
	BA-4		3 ø 16		3 ø 16		1	
B	BB-1	400	3ø12+1ø10	4.2 t/cm²	2 ø 8	4.2 t/cm²	0.24	10 ø 6
	BB-2		3ø12+1ø10		2ø8+1ø10		0.427	
	BB-3		3ø12+1ø10		2ø10+1ø12		0.647	
	BB-4		3ø12+1ø10		3ø12+1ø10		1	
C	BC-1	395	3ø12+1ø10	4.4 t/cm²	2 ø 8	4.4 t/cm²	0.24	10 ø 6
	BC-2		3ø12+1ø10		2ø8+1ø10		0.427	
	BC-3		3ø12+1ø10		2ø10+1ø12		0.647	
	BC-4		3ø12+1ø10		3ø12+1ø10		1	

Table (2) Curvatures and the Corresponding Ductility Factor

Group No.	Beam No.	Curvature at Py	Curvature at Pu	Curvature at .8Pu	Max. Ductility factor	Fin. Ductility factor
A	BA-1	0.01361	0.06	0.4627	4.41	34
	BA-2	0.0132	0.1527	1.0404	11.57	79
	BA-3	---	---	---	---	---
	BA-4	0.013	0.4458	---	27	---
B	BB-1	0.0198	0.098	0.3458	4.95	17
	BB-2	0.01905	0.3009	0.5276	15.8	27
	BB-3	0.01846	0.761	1.375	41.22	74
	BB-4	0.018	1.0837	1.697	60	94
C	BC-1	0.025	0.1084	0.3157	4.34	12.6
	BC-2	0.023	0.2159	0.34715	9.4	15
	BC-3	0.021	0.3445	---	16.4	---
	BC-4	0.02008	0.3212	---	16	---

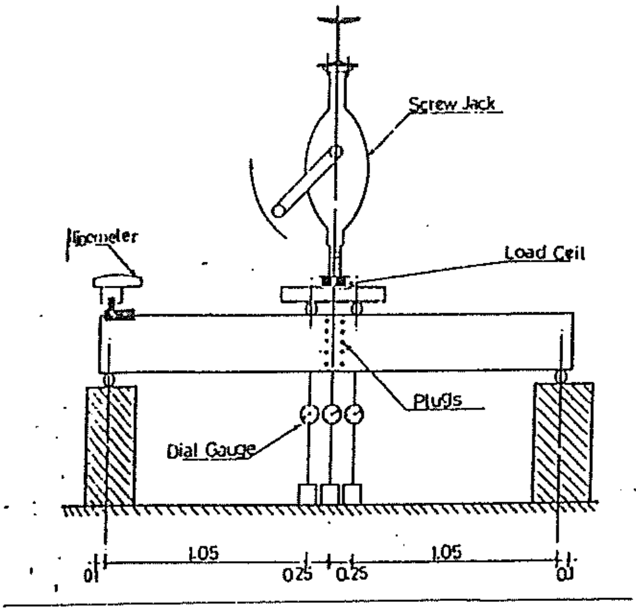


Fig. (1) Distribution of dial gauges and demic points.

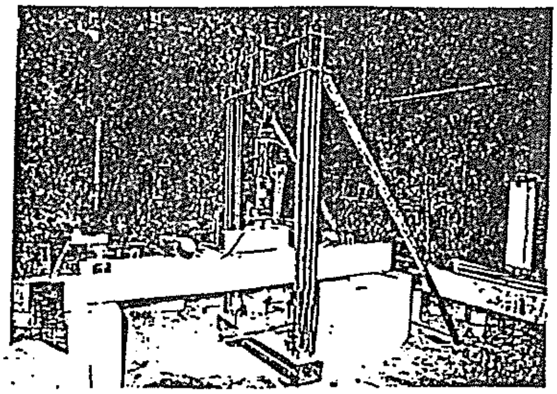


Fig. (2) The Mechanism of loading.

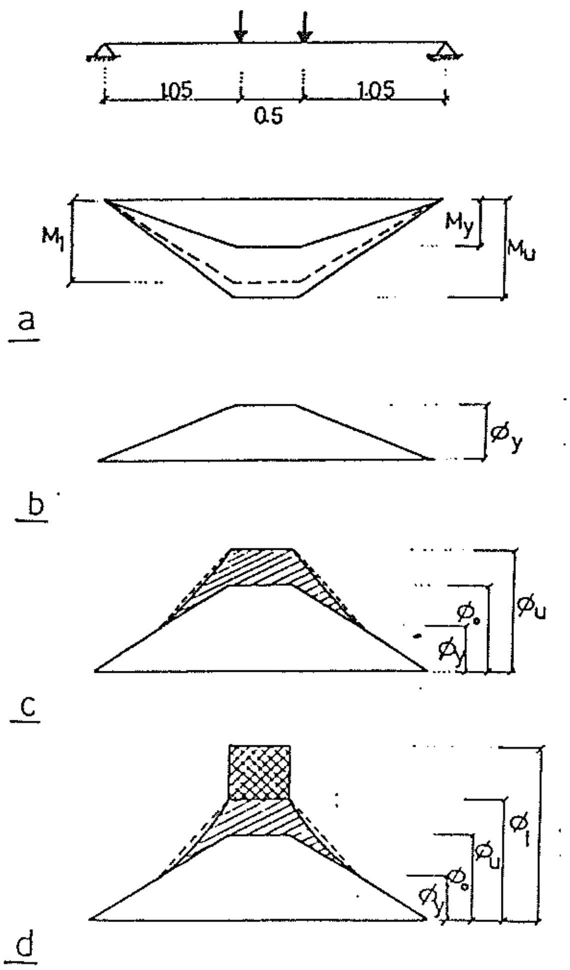


Fig.(3) The distribution of curvature with different levels of moment.

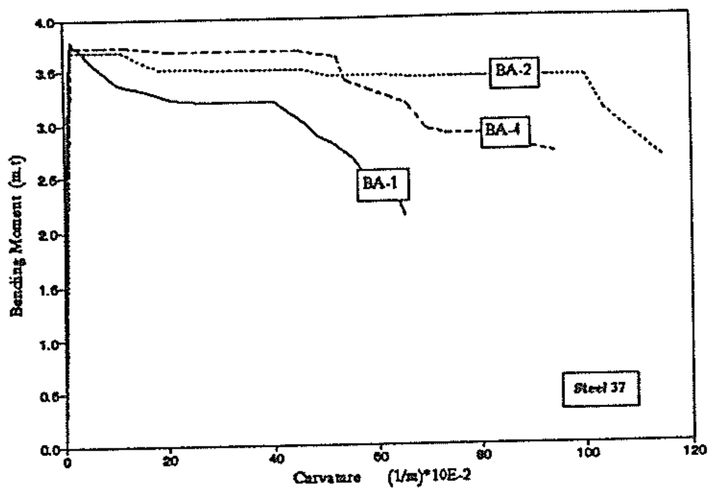


Fig. (4) Moment - Curvature diagrams for beams of group [A]

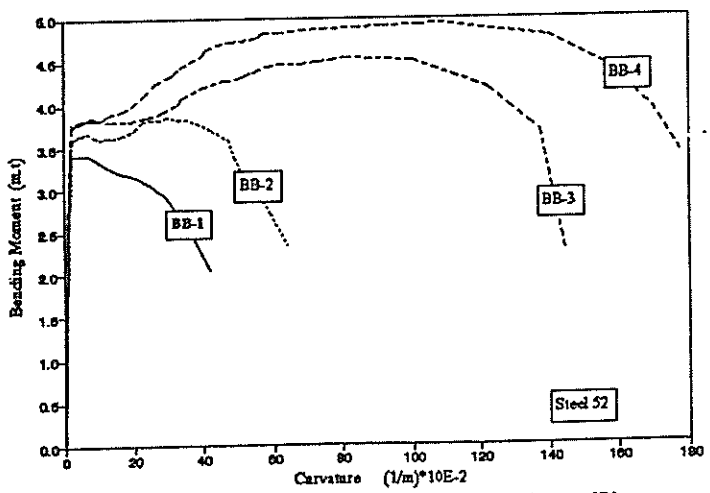


Fig. (5) Moment - Curvature diagram for beams of group [B]

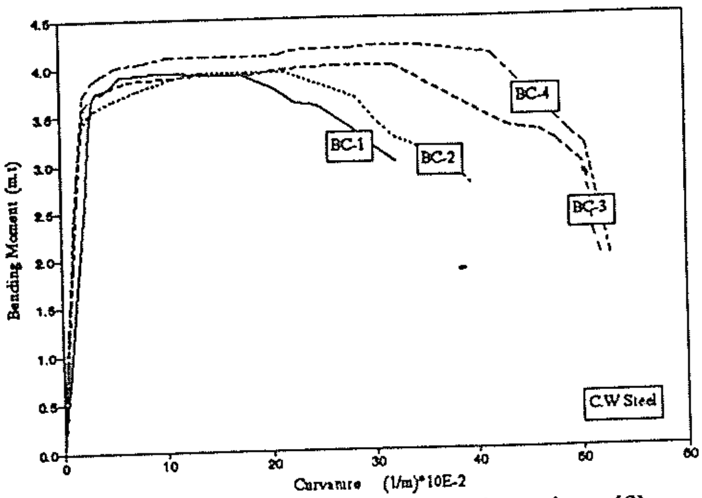


Fig. (6) Moment - Curvature diagram for beams of group [C]

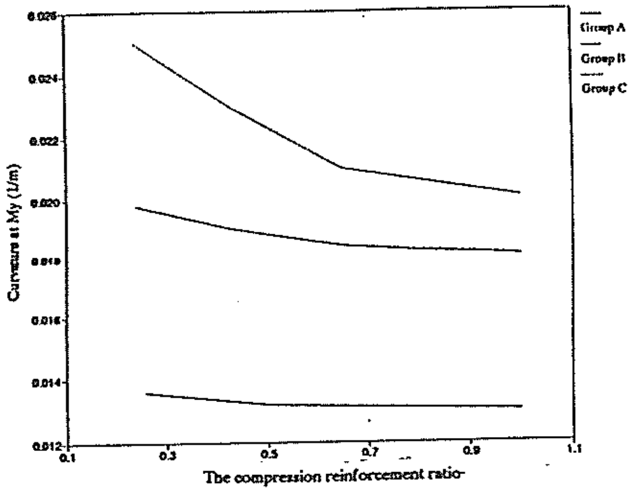


Fig. (7) Relasionship between max. Curvature and the compression reinforcement ratio

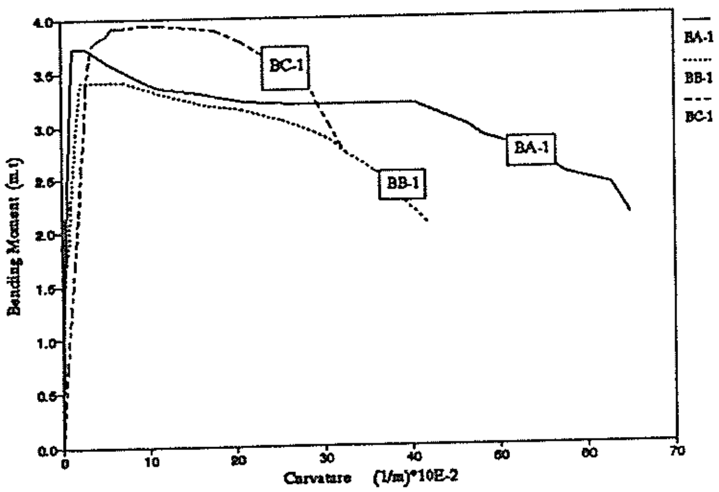


Fig. (8) Comparison of Moment - Curvature diagrams for beams BA-1 , BB-1 & BC-1

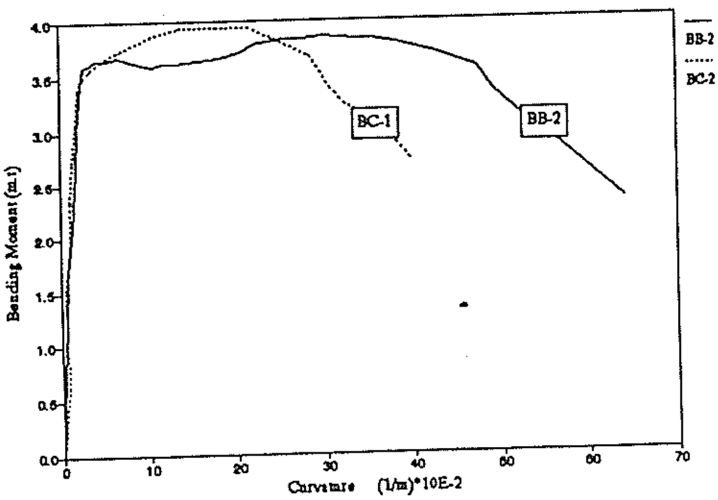


Fig. (9) Comparison of Moment - Curvature diagrams for beams BB-2 & BC-2

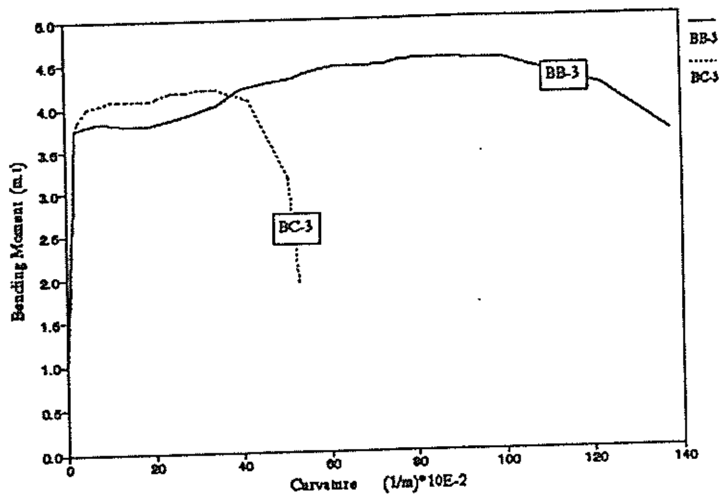


Fig. (10) Comparison of Moment - Curvature diagrams for beams BB-3 & BC-3

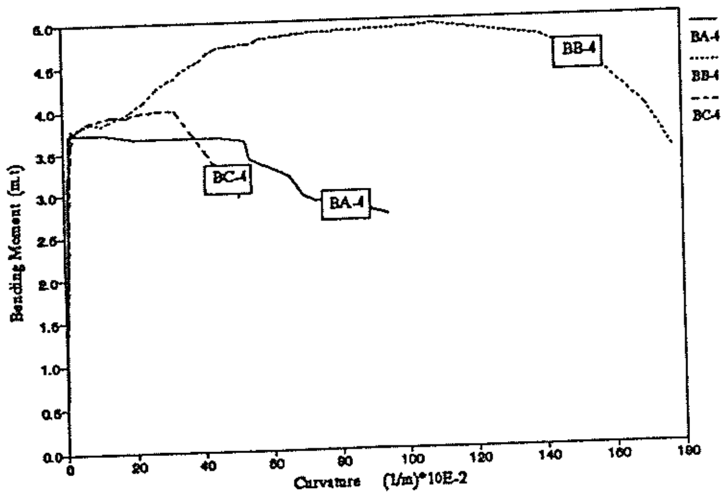
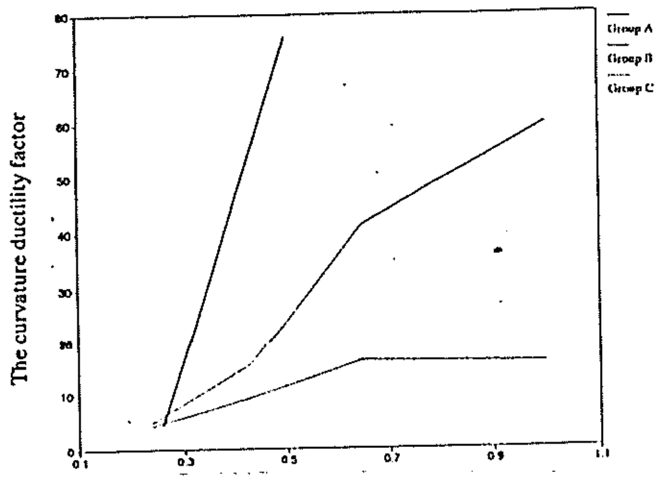


Fig. (11) Comparison of Moment - Curvature diagrams for beams BA-4 , BB-4 & BC-4



The compression reinforcement ratio

Fig. (12) relasionship between max. curvature ductility factor and the compression reinforcement ratio.